

Electromagnetic Waves

ELECTROMAGNETIC WAVES

Section - 1

We have seen that electric current produces magnetic field. We have also studied that a time varying magnetic field induces electric field. J.C. Maxwell proposed that a time-varying electric field also generates a magnetic field. He formulated a set of equations that form the basic laws of electromagnetism. Maxwell's equations predicted the existence of electromagnetic waves, which are self-sustaining oscillations of electric and magnetic fields in free space. The speed of these waves turned out to be very close to the speed of light ($3 \times 10^8 \text{ m/s}$). It was concluded that light is an electromagnetic wave.

Generation of Electromagnetic Waves :

Static charges produce only electrostatic field. And charges in uniform motion (steady currents) produce magnetic field that is not time varying.

Accelerated charges radiate electromagnetic waves.

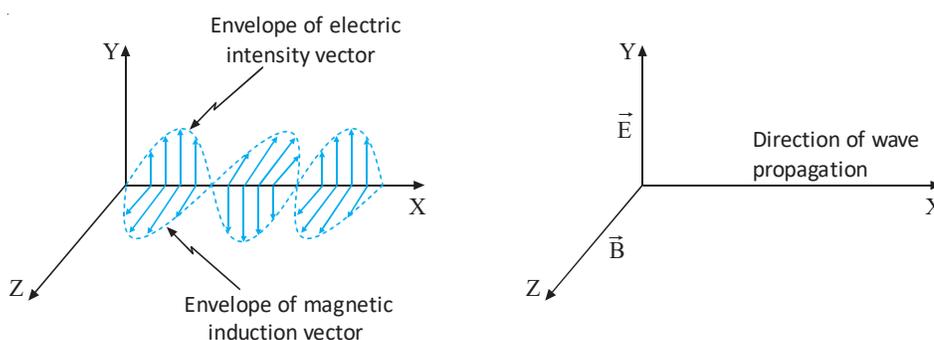
An oscillating charge will produce an oscillating electric field in space. This oscillating electric field gives rise to oscillating magnetic field in space, which in turn is a source of oscillating electric field and so on.

\therefore Oscillating electric and magnetic fields regenerate each other as the wave propagates through space.

Source of Electromagnetic Wave—Oscillating charge

Frequency of Electromagnetic Wave = Frequency of Oscillation of charge

Representation of Electromagnetic Wave :



Nature of Electromagnetic Waves

1. Electromagnetic waves are transverse in nature. These waves do not require any material medium for propagation and are also referred to as non-mechanical waves.

2. The electric and magnetic fields always oscillate in mutually perpendicular directions, and are always in phase. If E and B denote the magnitudes of the fields at any instant, and E_0 and B_0 denote their maximum magnitudes (i.e. their amplitudes), then

$$\frac{E}{B} = \frac{E_0}{B_0} = c_M$$

Here, c_M is the speed of light in the medium in which the electromagnetic wave is propagating.

3. If the direction of the electric field and the magnetic field of an electromagnetic wave is given by the unit vectors \hat{E} and \hat{B} , then the direction of propagation of the wave is along the unit vector $\hat{S} = \hat{E} \times \hat{B}$.

It follows that $\hat{E} = \hat{B} \times \hat{S}$ and $\hat{B} = \hat{S} \times \hat{E}$

(These three unit vectors \hat{S} , \hat{E} and \hat{B} form a cycle just like the unit vectors \hat{i} , \hat{j} and \hat{k})

The E -field, the B -field and the direction of propagation are all mutually perpendicular.

Example :

The E -field and the B -field of an EM wave propagating in the +X direction can be:

$$\vec{E} = [E_0 \sin(kx - \omega t + \phi)] \hat{j} ; \quad \vec{B} = [B_0 \sin(kx - \omega t + \phi)] \hat{k}$$

Notice that \vec{E} and \vec{B} are mutually perpendicular, in phase and the direction of propagation (i.e. +X) is given by $\hat{S} = \hat{E} \times \hat{B}$.

Here,

- (i) ω is the angular frequency of the wave (SI unit-radians per second (rad/s)).

$$\omega = 2\pi\nu$$

ν (pronounced like “new”) is the frequency of the wave (SI unit-Hertz (Hz)).

- (ii) k is the wave number (SI unit- m^{-1}).

$$k = \frac{2\pi}{\lambda}$$

λ is the wavelength (SI unit- m).

- (iii) The speed of the wave (c) is given by $c = \nu\lambda = \frac{\omega}{k}$

4. The speed of electromagnetic waves in a medium (c_M) depends on the electric and magnetic properties of the medium, in particular on the electric permittivity (ϵ) and magnetic permeability (μ) of the medium.

$$c_M = \frac{1}{\sqrt{\epsilon\mu}}$$

So, the speed of light in vacuum is given by: $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

And, it follows that

$$c_M = \frac{c}{\sqrt{\epsilon_r\mu_r}} \quad \text{and} \quad n_M = \frac{c}{c_M} = \sqrt{\epsilon_r\mu_r}$$

Here, ϵ_r and μ_r are the relative electric permittivity and the relative magnetic permeability of the medium respectively and n_M is the refractive index of the medium.

5. Electromagnetic waves carry energy in the form of electric and magnetic fields. The energy density (i.e. the energy contained per unit volume) (SI unit of energy density is J/m^3) in an infinitesimal neighbourhood of a point in vacuum where the magnitude of electric and magnetic field is E and B is given by:

$$U = U_E + U_B = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

Also, since $\frac{E}{B} = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$, $U_E = U_B$

This means that in the neighbourhood of any point through which an electromagnetic wave is passing, half of the total energy contained in a unit volume is due to the electric field and the other half is due to the magnetic field.

Now, the average energy density in the neighbourhood of a point is given by:

$$U_{AVG} = (U_E)_{AVG} + (U_B)_{AVG} = \frac{1}{2} \epsilon_0 (E^2)_{AVG} + \frac{1}{2\mu_0} (B^2)_{AVG} = \frac{1}{4} \epsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2$$

(We know that since E^2 and B^2 are functions of the type \sin^2 (or \cos^2), their average values are $\frac{1}{2}$ times their maximum values E_0 and B_0)

We also know that $E_0 = B_0 c = \frac{B_0}{\sqrt{\epsilon_0 \mu_0}}$

Therefore, $U_{AVG} = \frac{1}{2} \epsilon_0 E_0^2 = \frac{1}{2\mu_0} B_0^2$

(All of the above is true for a wave travelling in a medium other than vacuum as well, except that ϵ_0 must be replaced by ϵ , μ_0 must be replaced by μ and c must be replaced by c_M , the speed of light in the medium)

6. The intensity of an electromagnetic wave is defined as the energy passing through a unit area perpendicular to the direction of propagation per unit time. Its SI unit is W/m^2 .



Consider this cylindrical volume with cross-sectional area A and length $c\Delta t$ along x axis.

The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c .

Energy contained $U = U_{AVG} (c\Delta t)A$

$$\text{Intensity } I = \frac{U}{A\Delta t} = U_{AVG} (c)$$

$$= \frac{1}{2} \epsilon_0 E_0^2 c = \frac{1}{2\mu_0} B_0^2 c$$

Here, E_0 and B_0 are the amplitudes of the electric and magnetic field in vacuum.

In a medium, the intensity of the wave is given by:

$$I_M = \left(\frac{1}{2} \epsilon E_0^2 \right) c_M = \left(\frac{1}{2\mu} B_0^2 \right) c_M$$

In the above formula, E_0 and B_0 are the amplitudes of the electric and magnetic field in the medium and c_M is the speed of light in the medium.

7. Momentum and Radiation Pressure

Like all other waves, electromagnetic waves also carry energy and momentum.

When electromagnetic waves strike a surface, they exert a pressure on it, known as radiation pressure.

If the energy transferred to the surface in time t is U , then the magnitude of momentum delivered to the surface is :

$$\Rightarrow p = \frac{U}{c} \quad (\text{For complete absorption})$$

$$\Rightarrow p = \frac{2U}{c} \quad (\text{For complete reflection})$$

Illustration - 1

The oscillating magnetic field in a plane electromagnetic wave is given by:

$$B_y = (8 \times 10^{-6}) \sin [2 \times 10^{11} t + 300 \pi x] T$$

- (i) Calculate the wavelength of the electromagnetic wave.
 (ii) Write down the expression for the oscillating electric field.

SOLUTION :

Given equation is

$$B_y = (8 \times 10^{-6}) \sin [2 \times 10^{11} t + 300 \pi x] T$$

Comparing the given equation with the equation of magnetic field varying sinusoidally with x and t

$$B_y = B_0 \sin \left(\frac{2\pi x}{\lambda} + \frac{2\pi t}{T} \right)$$

$$\text{We get, } \frac{2\pi}{\lambda} = 300\pi$$

$$\therefore \lambda = \frac{2}{300} = 0.0067 \text{ m and } B_0 = 8 \times 10^{-6} T$$

- (i) Wavelength of the electromagnetic wave

$$\lambda = 0.0067 \text{ m}$$

$$\begin{aligned} \text{(ii) } E_0 &= cB_0 = 3 \times 10^8 \times 8 \times 10^{-6} \\ &= 24 \times 10^2 = 2400 \text{ Vm}^{-1} \end{aligned}$$

\(\therefore\) The required expression for the oscillating electric field is:

$$\begin{aligned} E_Z &= E_0 \sin \left(\frac{2\pi x}{\lambda} + \frac{2\pi t}{T} \right) \\ &= 2400 \sin (300 \pi x + 2 \times 10^{11} t) \text{ V / m} \end{aligned}$$

Illustration - 2

The electric field of an electromagnetic wave propagating in a medium is given by:

$$\vec{E} = 100 \sin \left((2\pi \times 10^4) z + (5\pi \times 10^{12}) t + \frac{\pi}{4} \right) \hat{i} \quad (\text{with all quantities in SI units})$$

- (i) Find the wavelength and the frequency of the wave.
 (ii) Find the speed of propagation of the wave and hence find the refractive index of the medium.
 (iii) Write the expression for the magnetic field of the electromagnetic wave.

SOLUTION :

Comparing the given equation with $\vec{E} = E_0 \sin \left(kz + \omega t + \frac{\pi}{4} \right) \hat{i}$,

We can see that the wavelength, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi \times 10^4} = 10^{-4} \text{ m}$

The frequency, $\nu = \frac{\omega}{2\pi} = \frac{5\pi \times 10^{12}}{2\pi} = \frac{5}{2} \times 10^{12} \text{ Hz}$

and the wave speed, $c_M = \nu\lambda = \frac{5}{2} \times 10^8 \text{ m/s}$

Therefore, the refractive index of the medium, $n_M = \frac{c}{c_M} = \frac{3 \times 10^8}{\frac{5}{2} \times 10^8} = \frac{6}{5}$

We can see that the direction of propagation is $\hat{S} = -\hat{k}$. Therefore, the direction of the magnetic field must be $\hat{B} = \hat{S} \times \hat{E} = (-\hat{k}) \times \hat{i} = -\hat{j}$.

Also, the amplitude of the magnetic field,

$$B_0 = \frac{E_0}{c_M} = \frac{100}{\frac{5}{2} \times 10^8} = 4 \times 10^{-7} \text{ T}$$

So, the magnetic field is given by $\vec{B} = (4 \times 10^{-7}) \sin\left((2\pi \times 10^4)z + (5\pi \times 10^{12})t + \frac{\pi}{4}\right) (-\hat{j})$

Illustration - 3 Electromagnetic waves travel in a medium with a speed of $2 \times 10^8 \text{ m/s}$. The relative permeability of the medium is 1. Find its relative permittivity.

SOLUTION :

Given : $c = 2 \times 10^8 \text{ m/s}$, $\mu_r = 1$

Speed of electromagnetic waves in medium

$$c_M = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}} = \frac{c_0}{\sqrt{\mu_r \epsilon_r}}; \epsilon_r = \frac{c_0^2}{c_M^2 \mu_r} = \frac{(3 \times 10^8)^2}{(2 \times 10^8)^2 \times 1} = 2.25$$

Illustration - 4 Light with an energy flux of 18 W/cm^2 falls on a completely absorbing surface at normal incidence. If the surface has an area of 20 cm^2 , find the average force exerted on the surface during a 30 minute time span.

SOLUTION :

The total energy falling on the surface is :

$$U = (18 \text{ W/cm}^2) \times (20 \text{ cm}^2) \times (30 \times 60) = 6.48 \times 10^5 \text{ J}$$

Therefore, the total momentum delivered (for complete absorption) is :

$$p = \frac{U}{c} = \frac{6.48 \times 10^5 \text{ J}}{3 \times 10^8 \text{ m/s}} = 2.16 \times 10^{-3} \text{ kg m/s}$$

The average force exerted on the surface is :

$$F = \frac{p}{t} = \frac{2.16 \times 10^{-3}}{0.18 \times 10^4} = 1.2 \times 10^{-6} \text{ N}$$

IN-CHAPTER EXERCISE-A

Choose the correct alternative. Only one choice is correct :

- A plane electromagnetic wave, $E_z = 50 \cos(12 \times 10^8 t + 8x)$ V/m propagates in a medium of refractive index :
 (A) 1.5 (B) 2.0 (C) 2.4 (D) 4.0
- Electromagnetic waves are produced by :
 (A) accelerated charges (B) static charges
 (C) charges in uniform motion (D) All of these
- A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, $\vec{E} = 6.3 \hat{j}$ V/m. What is the magnetic field at this point ?
 (A) $2.1 \times 10^{-8} \hat{j} T$ (B) $6.3 \times 10^{-8} \hat{j} T$ (C) $2.1 \times 10^{-8} \hat{k} T$ (D) $-6.3 \times 10^{-8} \hat{k} T$

For Questions 4 - 5

In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 Vm^{-1} .

- What is the wavelength of the wave ?
 (A) $1.5 \times 10^{-2} \text{ m}$ (B) $2.5 \times 10^{-2} \text{ m}$ (C) $6.6 \times 10^{-3} \text{ m}$ (D) $2.5 \times 10^{-3} \text{ m}$
- What is the amplitude of the oscillating magnetic field ?
 (A) $2.7 \times 10^{-7} T$ (B) $1.6 \times 10^{-7} T$ (C) $1.6 \times 10^{-8} T$ (D) $2.7 \times 10^{-8} T$
- Which of the following statement is false for the properties of electromagnetic waves ?
 (A) Both electric and magnetic field vectors attain the maxima and minima at the same place and same time.
 (B) The energy in electromagnetic wave is divided equally between electric and magnetic field vectors.
 (C) Both electric and magnetic field vectors are parallel to each other and perpendicular to the direction of propagation of wave.
 (D) These waves do not require any material medium for propagation.
- When light energy of 60 W is incident normally on a plane mirror. The total momentum delivered is:
 (A) $20 \times 10^{-7} \text{ kg ms}^{-1}$ (B) $4 \times 10^{-7} \text{ kg ms}^{-1}$
 (C) 40 kg ms^{-1} (D) $20 \times 10^{-8} \text{ kg ms}^{-1}$
- Radiations of intensity 0.5 Wm^{-2} are striking a metal plate and getting completely absorbed. The pressure on the plate is :
 (A) $0.166 \times 10^{-8} \text{ Nm}^{-2}$ (B) $0.332 \times 10^{-8} \text{ Nm}^{-2}$
 (C) $0.111 \times 10^{-8} \text{ Nm}^{-2}$ (D) $0.083 \times 10^{-8} \text{ Nm}^{-2}$
- If μ_0 is the permeability of free space and ϵ_0 is the permittivity of free space, the speed of light in vacuum is given by:
 (A) $\sqrt{\mu_0 \epsilon_0}$ (B) $\sqrt{\frac{\mu_0}{\epsilon_0}}$ (C) $\sqrt{\frac{1}{\mu_0 \epsilon_0}}$ (D) $\sqrt{\frac{\epsilon_0}{\mu_0}}$
- The average value of electric energy density in an electromagnetic wave is : (E_0 is peak value)
 (A) $\frac{1}{2} \epsilon_0 E_0^2$ (B) $\frac{E_0^2}{2\epsilon_0}$ (C) $\epsilon_0 E_0^2$ (D) $\frac{1}{4} \epsilon_0 E_0^2$

DISPLACEMENT CURRENT

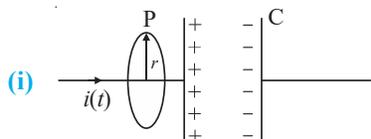
Section - 2

According to Ampere's circuital law : $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

where \vec{B} is the magnetic field related to steady current I .

We have studied that an electrical current produces magnetic field around it. Maxwell provided that for logical consistency, a changing electric field must also give rise to a changing magnetic field.

Consider a capacitor C which is being charged.

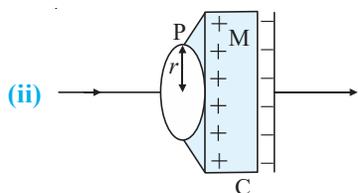


Let us find magnetic field at a point P outside the capacitor. For this, we consider a plane circular loop of radius r as shown. Magnetic field (B) is same in magnitude at all points of loop and magnetic field lines would be directed along the circumference of the loop.

Applying $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$ along loop, we get :

$$B(2\pi r) = \mu_0 i(t)$$

or
$$B = \frac{\mu_0 i(t)}{2\pi r}$$



Now let us consider a different surface with the same boundary. It is a pot like surface which does not touch the current, but has its bottom between the capacitor plates.

Applying Ampere's circuital law, we get :

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (0) \quad (\because i_{\text{enclosed}} = 0)$$

\therefore Application of Ampere's circuital law does not depend on shape of surface, results (i) and (ii) are inconsistent.

The electric flux ϕ_E through the surface between and parallel to the plates of the parallel plate capacitor, through which a time dependent current flows is given by: $\phi_E = |E|A = \left(\frac{1}{\epsilon_0} \frac{Q}{A}\right)A = \frac{Q}{\epsilon_0}$ (E : Electric field between the plates).

Now if the charge Q on the capacitor plate changes with time, there is a current $\left(i_d = \frac{dQ}{dt}\right)$.

We have,
$$\frac{d\phi_E}{dt} = \frac{d}{dt} \left(\frac{Q}{\epsilon_0} \right) = \frac{1}{\epsilon_0} \frac{dQ}{dt} \Rightarrow i_d = \epsilon_0 \left[\frac{d}{dt} (\phi_E) \right] \quad (i_d = \text{displacement current})$$

Conduction Current : Current carried by conductors due to flow of charges

Displacement Current : Displacement current is the equivalent current intercepting the surface S, which is due to the change in electric flux $\frac{d\phi_E}{dt}$. Displacement current exists through a surface when there is a time varying electric flux intercepted by the surface and the magnitude of this current is $\epsilon_0 \frac{d}{dt} \phi_E$.

Generalisation made by Maxwell

The source of magnetic field is not just the conduction electric current due to flowing charges, but also the time rate of change of electric field.

$$\begin{array}{ccccccc} \vec{i} & = & \vec{i}_c & + & \vec{i}_d & = & i_c + \epsilon_0 \left(\frac{d\phi_E}{dt} \right) \\ \text{total} & & \text{conduction} & & \text{displacement} & & \\ \text{current} & & \text{current} & & \text{current} & & \end{array}$$

Outside the capacitor plates, we have only conduction current i.e. $i = i_c$ and $i_d = 0$.

Inside the capacitor, there is no conduction current i.e. $i_c = 0$ and there is only displacement current i.e. $i = i_d$.

Time - dependent electric and magnetic fields give rise to each other.

Maxwell's Equations :

- $\int \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$ Gauss's Law for Electricity
- $\int \vec{B} \cdot d\vec{A} = 0$ Gauss's Law for Magnetism
- $\int \vec{E} \cdot d\vec{L} = -\frac{d\phi_B}{dt}$ Faraday's Law
- $\int \vec{B} \cdot d\vec{L} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$ Ampere - Maxwell Law

Illustration - 5 What is the instantaneous displacement current in space between plates of parallel plate capacitor of capacitor $1\mu F$ which is charging at the rate of 10^6 V/s.

SOLUTION :

$$\text{As } I_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 A \frac{d}{dt} (E) = \epsilon_0 A \frac{d}{dt} \left(\frac{V}{d} \right) = \frac{\epsilon_0 A}{d} \frac{dV}{dt} = C \frac{dV}{dt}; I_d = 10^{-6} \times 10^6 = 1A$$

Illustration - 6 A parallel-plate capacitor is being charged. Show that the displacement current across an area in the region between the plates and parallel to it is equal to the conduction current in the connecting wires.

SOLUTION :

The electric field between the plates is :

$$E = \frac{Q}{\epsilon_0 A}$$

where Q is the charge accumulated at the positive plate.

The flux of this field through the given area is

$$\phi_E = \frac{Q}{\epsilon_0}$$

The displacement current is

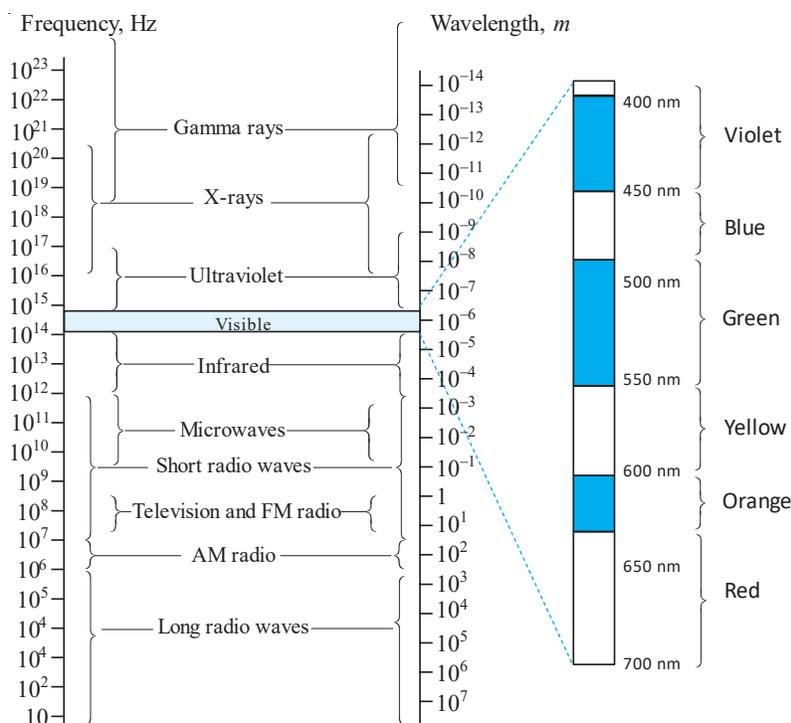
$$i_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{Q}{\epsilon_0} \right) = \frac{dQ}{dt} = i_c$$

where i_c is the conduction current.

ELECTROMAGNETIC SPECTRUM

Section - 3

Classification of electromagnetic waves is roughly based on the production and detection of waves.



The diagram shows electromagnetic spectrum, with common names for various part of it. The various regions do not have sharply defined boundaries.

Radio Waves :

- Produced by accelerated motion of charges in wires.
- Used in radio and TV communication system (AM, FM, TV, cellular phones).
- The frequency range of radio waves is roughly from 500 kHz to 1000 MHz.
- Cellular phones use radio waves to transmit voice communication in ultra high frequency (UHF) band.
- The frequency of AM is roughly from 530 kHz to 1710 kHz. The FM frequency is roughly from 88 MHz to 108 MHz. Frequency used for television broadcasting is from about 54 MHz to 890 MHz.
- Used in radio and TV communication system (AM, FM, TV, cellular phones).

Microwaves :

- Short wavelength waves with frequency in GHz (gigahertz) range.
- Produced by special vacuum tubes (Klystrons, magnet and gun diodes)
- Suitable for radar system used in aircraft navigation due to its shorter wavelengths.
- Radar is also used in speed guns to time fast balls, tennis serves and automobiles.
- In a microwave oven, frequency of microwaves are matched with frequency of resonance of water molecules. Thus, energy from waves is efficiently transferred to KE of water molecules, thereby heating food.

Infrared Waves :

- ▶ Produced by hot bodies and molecules. Infrared waves are also referred to as heat waves.
- ▶ Infrared band lies adjacent to low-frequency or long-wavelength end of visible spectrum.
- ▶ H_2O , CO_2 , NH_3 molecules readily absorb Infrared waves and heat up.
- ▶ Used in Infrared lamps in physiotherapy. Infrared waves also help maintain earth's warmth by greenhouse effect (Infrared radiations trapped by earth's atmosphere).
- ▶ Infrared detectors are used in satellites for military and agricultural purposes.
- ▶ Household remotes of electronic systems also use Infrared signals.

Visible Rays :

- ▶ Part of spectrum detected by human eye.
- ▶ The frequency of visible rays lies between $4 \times 10^{14} \text{ Hz}$ to $7 \times 10^{14} \text{ Hz}$. The wavelength of a visible rays lies between 400 nm to 700 nm .
- ▶ Helps us see the world around.
- ▶ Snakes can sense Infrared waves and insects can sense ultraviolet waves too.

Ultraviolet Rays :

- ▶ The wavelength of ultraviolet rays lies between 0.6 nm to 400 nm .
- ▶ Produced by special lamps and very hot bodies (esp. Sun)
- ▶ Most of ultraviolet radiation from sun is absorbed by the ozone layer in atmosphere. Exposure to ultraviolet can cause skin cancer.
- ▶ ultraviolet radiation is absorbed by glass. (\therefore No tanning occurs due to sunlight through glass windows)
- ▶ Welders wear special goggles/face masks with glass to protect eyes from ultraviolet radiations produced by welding arcs.
- ▶ Used in LASIK eye surgery for high precision (due to shorter wavelength).

X-rays :

- ▶ Lies beyond ultraviolet region of spectrum.
- ▶ Used in medical diagnosis and for treatment of some forms of cancer.
- ▶ The wavelength of X-rays lies between 10^{-4} nm to 10 nm .
- ▶ Can be produced by bombarding metal target by high energy electrons.
- ▶ X-rays can damage or destroy living tissues and organisms, so exposure must be careful and limited.

Gamma Rays :

- ▶ Upper of region of electromagnetic spectrum.
- ▶ The wavelength γ -rays lies between 10^{-10} m to less than 10^{-14} m .
- ▶ Produced in nuclear reactions and emitted by radioactive nuclei.
- ▶ Used in medicine to destroy cancer cells.

Different Types of Electromagnetic Waves			
Type	Wavelength Range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1m to 1mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1mm to 7nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye photocells Photographic film
Ultraviolet	400nm to 1nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film Geiger tubes Ionisation chamber
Gamma rays	< 10^{-3} nm	Radioactive decay of the nucleus	Photographic film Geiger tubes Ionisation chamber

Radiation in Atmosphere :

Electromagnetic radiation in atmosphere is mainly due to the sun. Sun sends electromagnetic waves of several wavelengths towards earth. A major part of this electromagnetic radiation is absorbed by earth. Visible light is just weakly absorbed. Most of the infrared radiation is absorbed and it heats the atmosphere. Ultraviolet radiation is absorbed by the ozone layer (present in upper part of stratosphere). Ozone layer also absorbs most of the radiations of lower wavelengths and protects us from their harmful effects. Ozone layer converts ultraviolet radiation to infrared radiation, which heats the earth's surface and atmosphere. The depletion of ozone layer is thus a cause of great concern to scientists and environmentalists.

IN-CHAPTER EXERCISE-B

Choose the correct alternative. Only one choice is correct :

- If ϕ_E denotes electric flux, the dimensions of the quantity $\epsilon_0 \frac{d\phi_E}{dt}$ are the same as :
 (A) Potential difference (B) Magnetic field
 (C) Current (D) Magnetic permeability
- Which of the following shows green house effect?
 (A) Ultraviolet rays (B) Infrared rays (C) X-rays (D) None of these
- All components of the electromagnetic spectrum in vacuum have the same :
 (A) energy (B) velocity (C) wavelength (D) frequency
- The frequencies of X-rays, γ -rays and ultraviolet rays are respectively a , b and c , then :
 (A) $a < b, b < c$ (B) $a > b, b > c$ (C) $a > b, b < c$ (D) $a < b, b > c$

5. A man can take pictures of those objects which are not fully visible to the eye using camera films acceptable to :
 (A) ultraviolet rays (B) sodium light (C) visible light (D) infrared rays
6. Infrared spectrum lies between :
 (A) radio wave and microwave region (B) microwave and visible region
 (C) visible and ultraviolet region (D) ultraviolet and X-ray

SUBJECTIVE SOLVED EXAMPLES

Example - 1 The amplitude of the magnetic field of an electromagnetic wave propagating in air is 10^{-7} T. The wave passes into another medium whose refractive index is 1.5 and whose relative permittivity (i.e. the dielectric constant) is 2 such that the intensity of the wave remains unchanged. Assume that the electric and magnetic properties of air are identical to vacuum.

(Permittivity of vacuum, $\epsilon_0 = 8.85 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2$) and Permeability of vacuum, $\mu_0 = 4\pi \times 10^{-7} \text{ m kg s}^{-2} \text{ A}^{-2}$)

Find :

- (i) the intensity of the wave
 (ii) the amplitude of the electric field of the wave, both in air and the other medium
 (iii) the amplitude of the magnetic field in the medium
 (iv) the relative permeability of the medium

SOLUTION :

We know that the intensity of an electromagnetic wave in vacuum is given by:

$$I = \frac{1}{2} \epsilon E_0^2 c = \frac{1}{2 \mu_0} B_0^2 c = \frac{1}{2(4\pi \times 10^{-7})} (10^{-7})^2 (3 \times 10^8) = \frac{15}{4\pi} \text{ W/m}^2$$

Amplitude of the electric field in air, $E_0 = B_0 c = (10^{-7})(3 \times 10^8) = 30 \text{ V/m}$

Let the amplitude of the electric field in the medium be E_M

Now, since the intensity of the wave is the same in both mediums, $I = I_M$

$$\Rightarrow \frac{1}{2} \epsilon_0 E_0^2 c = \frac{1}{2} \epsilon E_M^2 c_M$$

$$\Rightarrow E_M = \left(\sqrt{\frac{\epsilon_0}{\epsilon}} \right) \left(\sqrt{\frac{c}{c_M}} \right) E_0 = \left(\frac{1}{\sqrt{2}} \right) \left(\sqrt{\frac{3}{2}} \right) E_0 = \frac{\sqrt{3}}{2} E_0 = 15\sqrt{3} \text{ V/m}$$

Also, the speed of the wave in the medium, $c_M = \frac{c}{1.5} = 2 \times 10^8 \text{ m/s}$

Therefore, the amplitude of the magnetic field in the medium, $B_M = \frac{E_M}{c_M} = \frac{15\sqrt{3}}{2 \times 10^8} = \frac{15\sqrt{3}}{2} \times 10^{-8} \text{ T}$

We know that $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ and $c_M = \frac{1}{\sqrt{\epsilon \mu}}$

Therefore, $\mu_r = \frac{\mu}{\mu_0} = \left(\frac{c}{c_M} \right)^2 \left(\frac{\epsilon_0}{\epsilon} \right) = \left(\frac{3}{2} \right)^2 \left(\frac{1}{2} \right) = \frac{9}{8}$

Example - 2 The magnetic field of an Electromagnetic wave propagating in a medium is given by:

$$\vec{B} = 10^{-6} \cos\left(\left(10\pi \times 10^6\right)\left(\frac{-3x+4z}{5}\right) + \left(3\pi \times 10^{15}\right)t\right) \hat{j} \quad (\text{with all quantities in SI units})$$

- (i) Find the wavelength and the frequency of the wave.
 (ii) Find a unit vector along the direction of propagation of the wave.
 (iii) Write the expression for the electric field of the wave.

SOLUTION :

The general equation of the magnetic (or electric) field of an electromagnetic wave looks like:

$$\vec{B} = B_0 \cos(\vec{k} \cdot \vec{r} + \omega t + \phi) \hat{B}$$

Here, \vec{k} is the wave vector and its magnitude is $\frac{2\pi}{\lambda}$ and its direction is the same as the direction of propagation.

And, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, i.e. the position vector of a general point.

Also, \hat{B} is a unit vector along the direction in which the magnetic field oscillates.

We are given that $\vec{B} = 10^{-6} \cos\left(\left(10\pi \times 10^6\right)\left(\frac{-3x+4z}{5}\right) + \left(3\pi \times 10^{15}\right)t\right) \hat{j}$

Therefore, we can deduce that the magnetic field oscillates in the Y-direction, and

$$\vec{k} = \left(10\pi \times 10^6\right) \left(\frac{3\hat{i} - 4\hat{k}}{5}\right) \quad \text{and} \quad \omega = 3\pi \times 10^{15} \text{ rad/s}$$

So, since $\frac{2\pi}{\lambda} = 10\pi \times 10^6$, the wavelength of the wave is $\frac{2\pi}{10\pi \times 10^6} = 2 \times 10^{-7} \text{ m} = 200 \text{ nm}$

And, $\hat{S} = \left(\frac{3\hat{i} - 4\hat{k}}{5}\right)$ is a unit vector along the direction of propagation.

And, the frequency, $\nu = \frac{\omega}{2\pi} = \frac{3}{2} \times 10^{15} \text{ Hz}$

So, the wave speed, $c = \nu\lambda = \left(\frac{3}{2} \times 10^{15}\right) \left(2 \times 10^{-7}\right) = 3 \times 10^8 \text{ m/s}$

Therefore, the amplitude of the electric field,

$$E_0 = B_0 c = \left(10^{-6}\right) \left(3 \times 10^8\right) = 300 \text{ V/m}$$

Also, the direction along which the electric field oscillates is given by:

$$\hat{E} = \hat{B} \times \hat{S} = (\hat{j}) \times \left(\frac{3\hat{i} - 4\hat{k}}{5}\right) = -\left(\frac{4\hat{i} + 3\hat{k}}{5}\right)$$

So, the expression for the electric field of the wave is:

$$\vec{E} = 300 \cos\left(\left(10\pi \times 10^6\right)\left(\frac{-3x+4z}{5}\right) + \left(3\pi \times 10^{15}\right)t\right) \left[-\left(\frac{4\hat{i} + 3\hat{k}}{5}\right)\right]$$

Example - 3 Calculate the electric and magnetic fields produced by the radiation coming from a 100 W bulb at a distance of 3 m. Assume that the efficiency of the bulb is 2.5% and it is a point source.

SOLUTION :

The bulb, as a point source, radiates light in all directions uniformly. At a distance of 3m, the surface area of the surrounding sphere is :

$$A = 4\pi r^2 = 4\pi(3)^2 = 113 \text{ m}^2$$

The intensity at this distance is $I = \frac{\text{Power}}{\text{Area}} = \frac{100\text{W} \times 2.5\%}{113 \text{ m}^2} = 0.022 \text{ W/m}^2$

Half of this intensity is provided by the electric field and half by the magnetic field.

$$\frac{1}{2}I = (U_E)_{AVG} = \frac{1}{4}\epsilon_0 E_0^2 c \Rightarrow E_0 = \sqrt{\frac{2I}{\epsilon_0 c}}$$

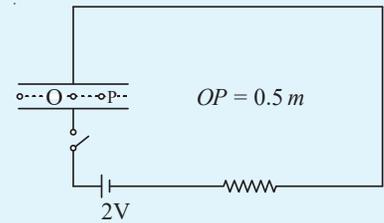
$$E_0 = \sqrt{\frac{2 \times 0.022}{(8.85 \times 10^{-12})(3 \times 10^8)}} \text{ V/m} = 4.07 \text{ V/m}$$

Electric field strength of light is fairly large.

$$B_0 = \frac{E_0}{c} = \frac{4.07 \text{ V/m}}{3 \times 10^8 \text{ m/s}} = 1.4 \times 10^{-8} \text{ T}$$

Note that although the energy in the magnetic field is equal to the energy in the electric field, the magnetic field strength is evidently very weak.

Example - 4 A parallel plate capacitor with circular plates of radius 1m has a capacitance of 1 nF. At $t = 0$, it is connected for charging in series with a resistor $R = 1 \text{ M}\Omega$ across a 2V battery (as shown). Calculate the magnetic field at a point P, halfway between the centre and the periphery of the plates, after $t = 10^{-3} \text{ s}$. (The charge on the capacitor at time t is $q(t) = CV [1 - e^{(-t/\tau)}]$, where the time constant τ is equal to CR)

**SOLUTION :**

The time constant of the CR circuit is $\tau = CR = 10^{-3} \text{ s}$. Then we have

$$\begin{aligned} q(t) &= CV \left[1 - e^{(-t/\tau)} \right] \\ &= 2 \times 10^{-9} \left[1 - e^{(-t/10^{-3})} \right] \end{aligned}$$

The electric field in between the plates at time t is $E = \frac{q(t)}{\epsilon_0 A} = \frac{q}{\pi \epsilon_0}$; $A = \pi(1)^2 \text{ m}^2 =$ area of the plates.

Consider a circular loop of radius 0.5 m parallel to the plates passing through P. The magnetic field B at all points on the loop is along the loop and of the same value. The flux through this loop is

$$= E \times \pi \times \left(\frac{1}{2}\right)^2 = \frac{\pi E}{4} = \frac{q}{4\epsilon_0}$$

The displacement current $i_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{1}{4} \frac{dq}{dt} = 0.5 \times 10^{-6} e^{(-1)}$ at $t = 10^{-3} s$. Now, applying Ampere-Maxwell law to the loop, we get :

$$B \times 2\pi \times \left(\frac{1}{2}\right) = \mu_0 (i_0 + i_d) = \mu_0 (0 + i_d) = 0.5 \times 10^{-6} \mu_0 e^{(-1)}$$

or $B = 0.74 \times 10^{-13} T$

Example - 5 *The intensity of the sunlight reaching the earth is $1380 W/m^2$. Assume this light to be a plane monochromatic wave. Find the amplitudes of electric and magnetic fields in this wave.*

SOLUTION :

$$\text{Intensity of wave} = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$\epsilon_0 = 8.85 \times 10^{-12}; E_0 = ?; c = 3 \times 10^8, I = 1380 W/m^2$$

$$1380 = \frac{1}{2} \times 8.85 \times 10^{-12} \times E_0^2 \times 3 \times 10^8$$

$$\Rightarrow E_0^2 = \frac{2 \times 1380}{8.85 \times 3 \times 10^{-4}} = 103.95 \times 10^4$$

$$\Rightarrow E_0 = 10.195 \times 10^2 = 1.02 \times 10^3 N/C$$

$$E_0 = B_0 c$$

$$\Rightarrow B_0 = E_0/c = \frac{1.02 \times 10^3}{3 \times 10^8} = 3.398 \times 10^{-5} = 3.4 \times 10^{-5} T$$

MISCELLANEOUS EXERCISE

Choose the correct options for each of the following questions. Questions marked with * may have more than one correct options.

- The electric field of an electromagnetic wave travelling through vacuum is given by the equation $E = E_0 \sin(kx - \omega t)$. The quantity that is independent of wavelength is :
 (A) k/ω (B) $k\omega$ (C) ω (D) k
- The magnetic field of an electromagnetic wave is given $B_y = 3 \times 10^{-7} \sin(10^3 x + 6.28 \cdot 10^{12} t) T$. The wavelength of the electromagnetic wave is :
 (A) 6.28 cm (B) 3.14 cm (C) 0.63 cm (D) 0.32 cm
- If a source is transmitting electromagnetic waves of frequency $8.196 \times 10^6 Hz$, then the wavelength of the electromagnetic waves transmitted from the source will be :
 (A) 5090 cm (B) 4050 cm (C) 4230 cm (D) 3660 cm
- The refractive index and the permeability of a medium are respectively 1.5 and $5 \times 10^{-7} Hm^{-1}$. The relative permittivity of the medium is nearly :
 (A) 6 (B) 15 (C) 81 (D) 10

5. The electromagnetic theory of light failed to explain :
 (A) Photoelectric effect (B) Polarisation
 (C) Diffraction (D) Interference
6. A perfectly reflecting mirror has an area of 1 cm^2 . Light energy is allowed to fall on it for 1 h at the rate of 10 W cm^{-2} . The force that acts on the mirror is :
 (A) $3.35 \times 10^{-8}\text{ N}$ (B) $6.7 \times 10^{-8}\text{ N}$ (C) $1.34 \times 10^{-7}\text{ N}$ (D) $2.4 \times 10^{-4}\text{ N}$
7. Electromagnetic waves can be deflected by :
 (A) Electric field only (B) Magnetic field only
 (C) Both (A) and (B) (D) None of these
8. A linearly polarised electromagnetic wave given as, $E = E_0 \sin(kz - \omega t)\hat{i}$ is incident from vacuum on the surface of glass normally. The reflected wave is given as:
 (A) $E_r = -E_0 \sin(kz + \omega t)\hat{i}$ (B) $E_r = -E_0 \cos(kz - \omega t)\hat{i}$
 (C) $E_r = -E_0 \cos(kz + \omega t)\hat{i}$ (D) $E_r = -E_0 \sin(kz - \omega t)\hat{i}$
9. The electric field intensity produced by the radiation coming from the 100 W bulb at a distance 1 m is E . The electric field at distance 2 m is:
 (A) E (B) $2E$ (C) $E/2$ (D) $E/4$
10. If \hat{E} and \hat{B} represent unit vectors along electric field vector and magnetic field vector. The unit vector along direction of propagation of electromagnetic wave is \hat{c} . Then,
 (A) $\hat{c} = (\hat{E} \cdot \hat{B})\hat{B}$ (B) $\hat{c} = (\hat{E} \times \hat{B})$ (C) $\hat{c} = (\hat{B} \times \hat{E})$ (D) None of these
- *11. Displacement current goes through the gap between the plates of a capacitor when the charge of the capacitor :
 (A) increases (B) decreases (C) does not change (D) is zero
12. Find the energy stored in a 60 cm length of a laser beam operating at 4 mW .
 (A) $8 \times 10^{-9}\text{ J}$ (B) $4 \times 10^{-9}\text{ J}$ (C) $8 \times 10^{-12}\text{ J}$ (D) $4 \times 10^{-12}\text{ J}$

ANSWERS TO IN-CHAPTER EXERCISES

A

1. B 2. A 3. C 4. A 5. B 6. C 7. B
 8. A 9. C 10. D

B

1. C 2. B 3. B 4. D 5. D 6. B

ANSWERS TO MISCELLANEOUS EXERCISE

1. A 2. C 3. D 4. D 5. A 6. B 7. D
 8. A 9. C 10. B 11. AB 12. C